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The estimation of surface thermal behavior of the working roll in hot rolling process

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Abstract—A two-dimensional inverse analysis utilizing the conjugate gradient method of minimization is applied to estimate the surface thermal behavior (i.e. heat flux and temperature) of a roll which was used in steel rolling mills with water cooling. No prior information is needed for the functional forms of timewise and spacewise variation of the surface heat flux and temperature. The transient temperature recordings inside the roll are modified from the existing experimental data and serve as the simulated experimental data for the inverse analysis. The results show that the estimation of surface heat flux and temperature becomes worse when the temperature measurements are taken too far away from the surface, due to a thin thermal layer. Once the surface heat flux is determined, the surface heat transfer coefficients can be calculated based on the Fourier law, provided that the ambient temperatures are known. After the surface heat transfer coefficients of the roll are determined, an evaluation of the cooling system of the roll can be performed.

1. INTRODUCTION

In metal forming processes, the roll has been widely used as the tool to deform metals. The processes involve extremely high pressure and temperature on the roll surface, which may produce a large thermal stress in the roll to cause roll wear or spalling. Therefore heat transfer information is very important for proper control of the thermal crown for shape control or the evaluation of direct cooling system of the roll, which could lead to increase in roll life [1]. Moreover, knowledge of the temperature in a rotating roll can also contribute to an understanding of the friction behavior encountered in various kinds of machines.

Considerable work has been done on studying the thermal behavior of the roll in rolling processes [2–6] based on a Lagrangian and a Eulerian coordinate system, respectively, but they deviate somewhat from the real operating situation, since the surface boundary condition of the roll is not known during the rolling processes. Recently, Tseng *et al.* [1] used the temperature measured from the roll surface as the boundary condition to calculate the temperature distribution inside the roll, as well as the surface heat transfer coefficients of the roll, and obtained a good estimation. However, the drawbacks are that installation of thermocouples on the roll surface is difficult, and they are easily burned out, even when used for a short period of time. Therefore, if the temperature is measured inside the roll and used to predict the surface thermal behavior of the roll, the disadvantages in ref. [1] can be avoided. This technique is the so-called 'inverse heat conduction problem'.

Various methods of solving the inverse heat conduction problems have been discussed in the text by Beck *et al.* [7]. During the past two decades, there has been a great interest in the use of the conjugate gradient method in iterative minimization procedures, applied to the solution of constrained and unconstrained problems involving linear and nonlinear equations. More recently, the method has been applied to the solution of the inverse heat conduction problem in a one-dimensional case [8–12], as well as in a two-dimensional case [13–16]. In this work the conjugate gradient method is adopted to estimate the surface heat flux of the roll in a two-dimensional inverse problem.

The method derives its basis from the perturbational principle and transforms the inverse problem into the solution of three problems, namely, the direct problem, the sensitivity problem and the adjoint problem, together with the gradient equation.

The commonly used steepest-descent method utilizes the direction of negative gradient of the functional as the search direction, while the conjugate gradient method uses a combination of the negative gradient of the functional and the previous descent

NOMENCLATURE

c heat capacity
$J(\theta, t)$ functional defined by equation (3)
$J'(\theta, t)$ gradient of the functional defined by equation (8c)
k thermal conductivity
P^k direction of descent at the kth iteration
$q(R_0, \theta, t)$ unknown surface heat flux
$T(r, \theta, t)$ estimated temperature
$\Delta T(r, \theta, t)$ sensitivity function satisfying the sensitivity problem defined by equation (4)
$Y(r, \theta, t)$ measured temperature.

Greek symbols
β^k search step size in going from q^k to q^{k+1}, in equation (9)
δ Dirac-delta function
δ_T thermal layer of a roll
ε convergence criteria
γ^k conjugate coefficient, defined by equation (11)
$\lambda(r, \theta, t)$ adjoint function (or Lagrange multiplier) satisfying the adjoint problem defined by equation (7)
ρ density
σ standard deviation of temperature measurement
ω angular velocity of a roll.

direction as the search direction. Obviously, methods which calculate each new direction of search as part of the descent direction at the last iteration are inherently more powerful than those in which the directions are assigned in advance.

In Sections 2–4, we present, respectively, the formulation of the direct, sensitivity, and adjoint problems for the determination for the functions $T(r, \theta, t)$, $\Delta T(r, \theta, t)$ and $\lambda(r, \theta, t)$, respectively. Once the functions $T(r, \theta, t)$, $\Delta T(r, \theta, t)$ and $\lambda(r, \theta, t)$ become available, the conjugate gradient method is applied as described in Section 5 to determine the timewise and spacewise variation of the boundary heat flux $q(R_0, \theta, t)$ and temperature $T(R_0, \theta, t)$.

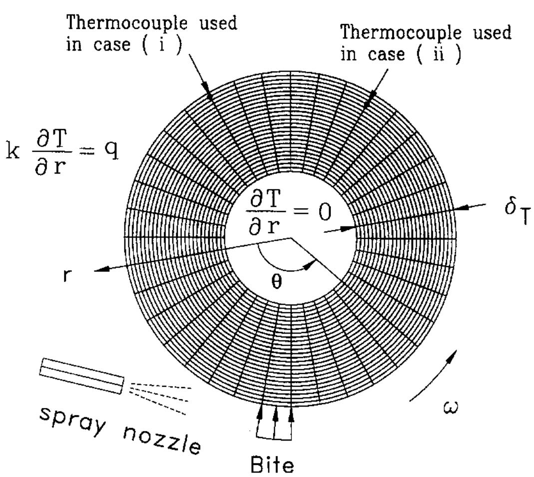


Fig. 2. Detail geometry and grid distribution of a work roll.

2. THE DIRECT PROBLEM

The initial temperature distributions of a roll are given as $T(r, \theta, 0) = f(r, \theta)$. For time $t > 0$, the rolling process takes place and the roll surface is subjected to a large prescribed heat flux within the bite region, while the heat will be dissipated into the ambience for the remaining surfaces. Figure 1 indicates the general illustration of the roll system and Fig. 2 shows the detail geometry and grid distribution of a work roll.

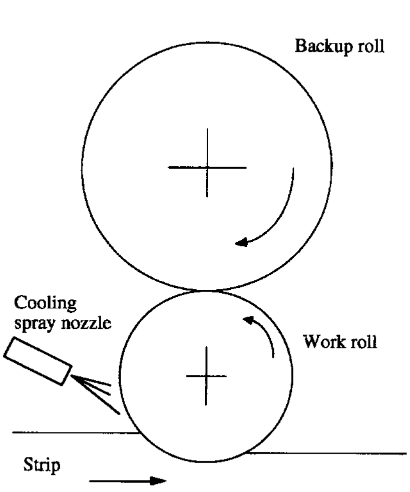


Fig. 1. General illustration of the roll system.

Using a Eulerian description and assuming constant thermal properties, the mathematical formulation governing the transient temperature fields $T(r, \theta, t)$ for a homogeneous and isotropic work roll is given by:

$$\alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) = \frac{\partial T}{\partial t} + \omega\frac{\partial T}{\partial \theta}$$

$$\text{in } R_i < r < R_0 \quad 0 < \theta < 2\pi \quad t > 0 \qquad (1a)$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = R_i \quad t > 0 \qquad (1b)$$

$$k\frac{\partial T}{\partial r} = q(\theta, t) \quad \text{at} \quad r = R_0 \quad t > 0 \qquad (1c)$$

$$T(r, 0, t) = T(r, 2\pi, t)$$

$$\text{at} \quad \theta = 0, \quad \text{and} \quad \theta = 2\pi \quad t > 0 \qquad \text{(1d)}$$

$$T(r,\theta,0) = f(r,\theta) \quad \text{for} \quad t = 0 \qquad \text{(1e)}$$

where r and θ are the radial and circumferential directions, respectively; ω is the angular velocity and α is the thermal diffusivity of the roll. The above parameters are assumed constant and known, while $q(\theta,t) = q(R_0,\theta,t)$ is the unknown boundary heat flux. Since the roll is rapidly rotating, all temperature variations are localized in a very thin layer near the surface: thus only a thin layer needs to be considered in the analysis. Therefore the interior boundary condition is taken as shown in equation (1b), where $R_0 - R_i = \delta_T$, and δ_T is taken as the depth of the steady-state skin thermal layer [17].

The Eulerian description used here will result in an elliptical-type governing equation having both conduction and convection terms, and the numerical solutions of this type will exhibit oscillation when the rotating speed is high [17]. Such a difficulty can be eliminated by using upwind differencing to approximate the convection term, but, in the mean time, an artificial diffusion α_p will be produced. The equivalent governing equation becomes

$$\alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + (\alpha+\alpha_p)\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \frac{\partial T}{\partial t} + \omega\frac{\partial T}{\partial \theta}$$

$$\text{in} \quad R_i < r < R_0 \quad 0 < \theta < 2\pi \quad t > 0 \qquad \text{(2a)}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = R_i \quad t > 0 \qquad \text{(2b)}$$

$$k\frac{\partial T}{\partial r} = q(\theta,t) \quad \text{at} \quad r = R_0 \quad t > 0 \qquad \text{(2c)}$$

$$T(r,0,t) = T(r,2\pi,t) \quad \text{at} \quad \theta = 0, \quad \text{and} \quad \theta = 2\pi \quad t > 0 \qquad \text{(2d)}$$

$$T(r,\theta,0) = f(r,\theta) \quad \text{for} \quad t = 0 \qquad \text{(2e)}$$

where $\alpha_p = \frac{1}{2}r^2\omega\Delta\theta[1-\omega(\Delta t/\Delta\theta)]$ is the artifical diffusion which depends on the circumferential and time mesh sizes as well as the angular velocity involved. The above direct problem can be solved by using alternating directional implicit (ADI) techniques.

The inverse analysis utilizing the conjugate gradient method requires the solution of direct, sensitivity and adjoint problems, together with the gradient equation. The developments of the sensitivity and adjoint equations and their solutions are discussed below.

3. THE SENSITIVITY PROBLEM

The solution of problem (2) with surface heat flux $q(\theta,t)$ unknown can be recast as a problem of optimum control, i.e. choose the control function $q(\theta,t)$ such that it minimizes the following functional:

$$J(q(\theta,t)) = \int_0^{2\pi}\int_0^{t_f}(T-Y)^2\,\mathrm{d}t\,\mathrm{d}\theta \qquad \text{(3)}$$

where $T(R_m,\theta,t)$ and $Y(R_m,\theta,t)$ are the estimated and measured temperatures at the positions (R_m,θ). If the estimation of $q(\theta,t)$ is available, the temperature T can be obtained from the solution of the direct problem at the specified measurement positions.

It is assumed that, when $q(\theta,t)$ undergoes an increment $\Delta q(\theta,t)$; then the temperature $T(r,\theta,t)$, changes by an amount $\Delta T(r,\theta,t)$. To construct the sensitivity problem satisfying the function $\Delta T(r,\theta,t)$, we replace T by $T+\Delta T$ and q by $q+\Delta q$ in the direct problem (2) and then subtract from it the original problem (2). The following sensitivity problem is obtained for the determination of the function $\Delta T(r,\theta,t)$:

$$\alpha\left(\frac{\partial^2 \Delta T}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta T}{\partial r}\right) + (\alpha+\alpha_p)\frac{1}{r^2}\frac{\partial^2 \Delta T}{\partial \theta^2} = \frac{\partial \Delta T}{\partial t} + \omega\frac{\partial \Delta T}{\partial \theta}$$

$$\text{in} \quad R_i < r < R_0 \quad 0 < \theta < 2\pi \quad t > 0 \qquad \text{(4a)}$$

$$\frac{\partial \Delta T}{\partial r} = 0 \quad \text{at} \quad r = R_i \quad t > 0 \qquad \text{(4b)}$$

$$k\frac{\partial \Delta T}{\partial r} = \Delta q(\theta,t) \quad \text{at} \quad r = R_0 \quad t > 0 \qquad \text{(4c)}$$

$$\Delta T(r,0,t) = \Delta T(r,2\pi,t)$$

$$\text{at} \quad \theta = 0, \quad \text{and} \quad \theta = 2\pi \quad t > 0 \qquad \text{(4d)}$$

$$\Delta T(r,\theta,0) = 0 \quad \text{for} \quad t = 0. \qquad \text{(4e)}$$

The above sensitivity problem (4) can then be solved using ADI techniques.

4. THE ADJOINT PROBLEM

To derive the adjoint equation we multiply equation (2a) with the adjoint function (or Lagrange multiplier) $\lambda(r,\theta,t)$, integrate the resulting expression over the total time t_f and the total circumferential domain $0 \leqslant \theta \leqslant 2\pi$, and then add this result to the functional given by equation (3). The following expression results:

$$J(q(\theta,t)) = \int_0^{2\pi}\int_0^{t_f}(T-Y)^2\,\mathrm{d}t\,\mathrm{d}\theta + \int_{R_i}^{R_0}\int_0^{2\pi}\int_0^{t_f} r\lambda \times \left[\alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + (\alpha+\alpha_p)\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} - \left(\frac{\partial T}{\partial t} + \omega\frac{\partial T}{\partial \theta}\right)\right]\mathrm{d}t\,\mathrm{d}\theta\,\mathrm{d}r. \qquad \text{(5)}$$

The variation $\Delta J(\theta,t)$ of equation (5) is then obtained by the perturbational principle [10–11] as follows:

$$\Delta J(\theta,t) = \int_{R_i}^{R_0}\int_0^{2\pi}\int_0^{t_f} 2\frac{r}{R_m}(T-Y)\Delta T\delta(r-R_m) \times \mathrm{d}t\,\mathrm{d}\theta\,\mathrm{d}r + \int_{R_i}^{R_0}\int_0^{2\pi}\int_0^{t_f} r\lambda \times \left[\alpha\left(\frac{\partial^2\Delta T}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta T}{\partial r}\right)+(\alpha+\alpha_p)\frac{1}{r^2}\frac{\partial^2 \Delta T}{\partial \theta^2} - \left(\frac{\partial \Delta T}{\partial t}+\omega\frac{\partial \Delta T}{\partial \theta}\right)\right] \mathrm{d}t\,\mathrm{d}\theta\,\mathrm{d}r \quad (6)$$

where $\delta(\cdot)$ is the Dirac-delta function and R_m are the positions of the thermocouples. The second triple integral term in equation (6) is integrated by parts, and the boundary conditions of the sensitivity problem defined by equation (4) are utilized to obtain the following adjoint problem:

$$\alpha\left(\frac{\partial^2\lambda}{\partial r^2}+\frac{1}{r}\frac{\partial\lambda}{\partial r}\right)+(\alpha+\alpha_p)\frac{1}{r^2}\frac{\partial^2\lambda}{\partial\theta^2}+\frac{\partial\lambda}{\partial t} + \omega\frac{\partial\lambda}{\partial\theta}+\frac{2}{\rho c R_m}(T-Y)\delta(r-R_m)=0$$

$$\text{in} \quad R_i < r < R_0 \quad 0<\theta<2\pi \quad t>0 \quad (7a)$$

$$\frac{\partial\lambda}{\partial r}=0 \quad \text{at} \quad r=R_i \quad t>0 \quad (7b)$$

$$\frac{\partial\lambda}{\partial r}=0 \quad \text{at} \quad r=R_0 \quad t>0 \quad (7c)$$

$$\lambda(r,0,t)=\lambda(r,2\pi,t) \quad \text{at} \quad \theta=0, \quad \text{and} \quad \theta=2\pi \quad t>0 \quad (7d)$$

$$\lambda(r,\theta,t_f)=0 \quad \text{for} \quad t=t_f. \quad (7e)$$

The adjoint problem (7) is different from the standard initial value problems in that the final time condition at time $t=t_f$ is specified instead of the customary initial condition $t=0$. However, the problem (7) can be transformed to an initial value problem by the transformation of the time variable as $\tau = t_f - t$. Then the standard ADI techniques can be used to solve the adjoint problem.

4.1. *Gradient equation*

After the above adjoint equation is obtained, the perturbation of functional still has one term left, it is

$$\Delta J(\theta,t) = \int_0^{2\pi}\int_0^{t_f} R_0\lambda(R_0,\theta,t)\Delta q(\theta,t)\,\mathrm{d}t\,\mathrm{d}\theta. \quad (8a)$$

Before obtaining the expression of gradient equation, we first introduce a notation of gradient of functional (the first Frechet derivative [18]). For $q=q(\theta,t)$, $\theta\in[0,2\pi]$ and $t\in[0,t_f]$, are the functions considered as elements of functional space $q\in L_2$-norm. If the functional increment can be presented as

$$\Delta J(\theta,t) = \int_0^{2\pi}\int_0^{t_f} J'(\theta,t)\Delta q(\theta,t)\,\mathrm{d}t\,\mathrm{d}\theta \quad (8b)$$

then $J'(\theta,t)$ is called a gradient of functional. Finally, the gradient equation becomes

$$J'(\theta,t) = R_0\lambda(R_0,\theta,t). \quad (8c)$$

5. INVERSE SOLUTION BY CONJUGATE GRADIENT METHOD

In this section an algorithm is presented for solving the two-dimensional inverse heat conduction problem described previously with the conjugate gradient method. The method is stable and converges quickly if some information is available for the final time condition of the unknown function $q(\theta,t_f)$, otherwise the inverse solutions at the final time will deviate from the exact solutions [8]. In this study, this is the case. In order to avoid such a difficulty, the modified conjugate gradient method [9] should be applied, or, for the purpose of simplicity, we can prolong the calculated time domain and then extract the inverse solutions just to the desired final time (i.e. the calculated time domain > the desired final time).

The following iterative procedure [10–12] is used for the determination of the surface heat flux:

$$q^{k+1} = q^k - \beta^k P^k \quad k=0,1,2,\ldots \quad (9)$$

where the direction of descent P^k is determined from the following relation:

$$P^k = J'^k + \gamma^k P^{k-1}. \quad (10)$$

Here P^{k-1} is its value of P at step $k-1$ and J'^k is the value of the gradient of the functional at step k. Although different definitions of the conjugate coefficient γ^k can be found in the standard texts on mathematics, we choose the form [15]

$$\gamma^k = \frac{\int_0^{2\pi}\int_0^{t_f}[J'^k(\theta,t)]^2\,\mathrm{d}t\,\mathrm{d}\theta}{\int_0^{2\pi}\int_0^{t_f}[J'^{k-1}(\theta,t)]^2\,\mathrm{d}t\,\mathrm{d}\theta} \quad \text{with} \quad \gamma^0=0. \quad (11)$$

The coefficient β^k, which determines the search step size in going from q^k to q^{k+1} in equation (9), is obtained by minimizing $J(q^{k+1})$ with respect to β^k, i.e.

$$\min_{\beta^k} J(q^{k+1}) = \min_{\beta^k}\int_0^{2\pi}\int_0^{t_f}[T(q^k-\beta^kP^k)-Y]^2\,\mathrm{d}t\,\mathrm{d}\theta. \quad (12a)$$

First, the Taylor series expansion is used to linearize the right-hand side of this expression in the form

$$\min_{\beta^k} J(q^{k+1}) = \min_{\beta^k} \times \int_0^{2\pi}\int_0^{t_f}[T(q^k)-\beta^k\Delta T(P^k)-Y]^2\,\mathrm{d}t\,\mathrm{d}\theta. \quad (12b)$$

Then equation (12b) is minimized by differentiating it with respect to β^k and equating it to zero. After rearrangement, the following expression is obtained for step size β^k:

$$\beta^k = \frac{\int_0^{2\pi}\int_0^{t_f}[\Delta T(P^k)(T-Y)]\,\mathrm{d}t\,\mathrm{d}\theta}{\int_0^{2\pi}\int_0^{t_f}\Delta T^2(P^k)\,\mathrm{d}t\,\mathrm{d}\theta}. \quad (13)$$

Once P^k is computed from equation (10) and β^k from equation (13), the iterative process defined by equation (9) can be applied to determine q^{k+1} until a specified stopping criterion is satisfied.

6. DISCREPANCY PRINCIPLE FOR STOPPING CRITERIA

If the problem involves no measurement errors, the traditional check condition specified as

$$J(q^{k+1}) < \varepsilon_1^2 \quad (14a)$$

where ε_1 is a small specified number. However, when the observed temperature data contain measurement errors, the inverse solution will tend to approach the perturbed input data, and the solution will exhibit oscillatory behavior as the number of iterations is increased [8]. Computational experience shows that it is advisable to use the discrepancy principle [8] for terminating the iteration process.

The discrepancy principle establishes the value of ε from equation (3) by assuming $(T-Y) \cong \sigma$, where σ is the standard deviation of the measurement error. This value of ε is then used as a stopping criteria, i.e.

$$J(q^{k+1}) = \int_0^{2\pi}\int_0^{t_f}\sigma^2\,\mathrm{d}t\,\mathrm{d}\theta \leqslant \varepsilon^2. \quad (14b)$$

7. THE ALGORITHM

The algorithm for the computational procedure of the iterative scheme starting from the kth iteration is summarized as

Step 1: $q^k(\theta, t)$ is available at the kth iteration, solve the direct problem given by equations (2) and compute $T(r, \theta, t)$;

Step 2: knowing $T(r, \theta, t)$ and the measured temperatures $Y(r, \theta, t)$, solve the adjoint problem defined by equations (7) and obtain the adjoint variables $\lambda(r, \theta, t)$;

Step 3: knowing $\lambda(r, \theta, t)$, compute the gradient of the functional, $J'(\theta, t)$, from equation (8c);

Step 4: knowing $J'(\theta, t)$, first compute γ^k from equation (11) and then compute the direction of descent P^k from equation (10);

Step 5: knowing the direction of descent P^k and setting $\Delta q^k = P^k$ [14], solve the sensitivity problem given by equations (4) and determine the sensitivity function $\Delta T(P^k)$;

Step 6: knowing $\Delta T(p^k)$, compute step size β^k from equation (13);

Step 7: knowing step size β^k, compute new heat flux $q^{k+1}(\theta, t)$ from equation (9);

Step 8: check if the stopping criteria given by equation (14b) is satisfied;

Step 9: if not, repeat the above calculational procedure until the stopping criteria given by equation (14b) is satisfied.

8. RESULTS AND DISCUSSIONS

To evaluate the validity of the present algorithm applied in a two-dimensional inverse calculation in predicting the surface heat flux of a roll, the temperature measurements inside the roll are needed, but such information is not available because no experiments were performed. Fortunately, Tseng provided a set of experiment data for the roll surface temperature as shown in Fig. 3(a), which was recorded with respect to time (time sequential information) at a specific surface location. The experimental set up is basically similar to ref. [1] and a thermocouple of type K is embedded in the surface to measure the temperature. The cooling water has a temperature of 16°C and spray pressure of 0.46 MPa at $\theta = 35°$, and the bite region is $\theta = 50$–$60°$.

However this temperature information needs to be the circumferential surface temperatures (spatial information) at a specific time to perform the inverse calculation. The technique used in ref. [1] is applied to convert the time sequential temperature information to the spatial information as shown in Fig. 3(b), where the number of cycles are one less than in Fig. 3(a) for the reason stated in ref. [1]. The following physical quantities are obtained from Tseng's experimental devices and used in the calculations:

$$k = 48.1\ \mathrm{W\,m^{-1}\,°C^{-1}} \quad c = 490\ \mathrm{J\,m^{-1}\,°C^{-1}}$$

$$\rho = 7830\ \mathrm{kg\,m^{-3}} \quad \omega = 155.6\ \mathrm{rpm}$$

$$R_i = 0.35\ \mathrm{m} \quad R_0 = 0.355\ \mathrm{m}$$

$$\delta_T = 0.005\ \mathrm{m}.$$

The reason for choosing above value for R_i is because the steady-state thermal layer thickness calculated based on ref. [17] is about 5 mm. The simulation of the interior measured temperatures is then calculated using the surface temperature information, which was obtained from Tseng, as the boundary condition, together with a proper initial condition $f(r, \theta)$. Since the first cycle at $t = 0$ in Fig. 3(b) is actually the second cycle in Fig. 3(a), so the initial temperature distribution $f(r, \theta)$ can be simulated by using the first cycle of the modified temperature [$t = 0$ in Fig. 3(b)] as the boundary condition and $t_i = 20°C$ as the initial condition (assuming the roll is initially at 20°C). Then calculate the temperature distribution for one cycle (the period of one cycle is 0.3856 s) and the temperature distribution at the end of the calculated cycle

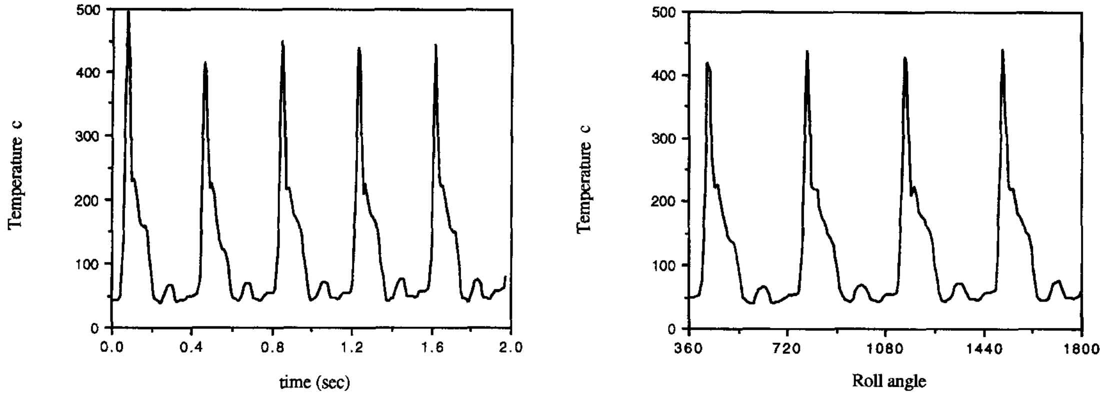


Fig. 3. (a) Measured surface temperature (time sequential information). (b) Measured surface temperature (spatial information).

is adopted as the initial condition $f(r, \theta)$ in the direct problem calculation. Finally, the temperature distribution at some specific position where the thermocouples were assumed installed can be obtained and used as the simulated measured temperatures to predict the surface heat flux in the inverse analysis.

Since the surface temperature shown in Fig. 3(b) looks the same, the surface heat flux for only one cycle of operation is estimated ($t = 0.3856$ s is needed). However, to avoid the difficulty in estimating $q(R_0, \theta, t_f)$ at the final time, we should perform the inverse calculation up to $t = 0.4156$ s (more than one cycle of operation), and then the inverse solutions are extract to $t = 0.3856$ s. The inverse calculation in such a manner can eliminate the difficulty, as was stated before.

In this study we will examine the inverse solution for different measured positions, i.e. the measured positions are assumed to be at (i) 0.2 and (ii) 1 mm from the roll surface, respectively. We also assume the standard deviation of the measurement errors, σ, are within $0.1 < \sigma < 0.3$, and therefore, for both above cases, the number of iterations can be taken as 35 based on the discrepancy principle. The initial guess of q is taken as zero in all the calculations that were shown in this section. Now let us illustrate some results obtained from numerical experiments for two different measurement positions.

8.1. *The measured position is* 0.2 *mm from the roll surface*

Figure 2 shows the geometry and grid distribution of a roll. The bold point in Fig. 2 indicates the position where the thermocouple is installed. Figure 4(a) and (b) shows the comparison between the measured and estimated temperature at $R = 0.3548$ (i.e. 0.2 mm from the roll surface) and 0.355 m (i.e. the roll surface), respectively, for $t = 0.3856$ s (i.e. at the end of one cycle), while Figs. 5 and 6 show the three-dimensional plot of the measured and estimated temperature at $R = 0.3548$ and 0.355 m, respectively.

It should be noted that the simulated measured temperature distribution at $R = 0.3548$ m is calculated based on the surface measured temperature (which was obtained from Tseng) and the estimated temperatures at $R = 0.3548$ and 0.355 m are calculated from the direct problem (2) by using the estimated surface feat flux as the boundary condition. This implies that they are obtained from different bases. Figures 4(a) and 5 reveal that the estimated and measured temperatures at $R = 0.3548$ m are in a very good agreement and similar results are obtained at

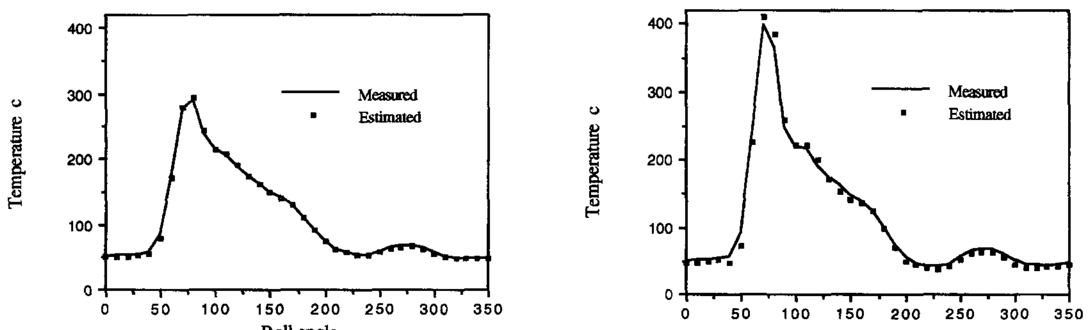


Fig. 4. (a) The estimated and measured temperatures located at 0.2 mm from the roll surface at $t = 0.3856$ s for case (i). (b) The estimated and measured temperatures located on the roll surface at $t = 0.3856$ s for case (i).

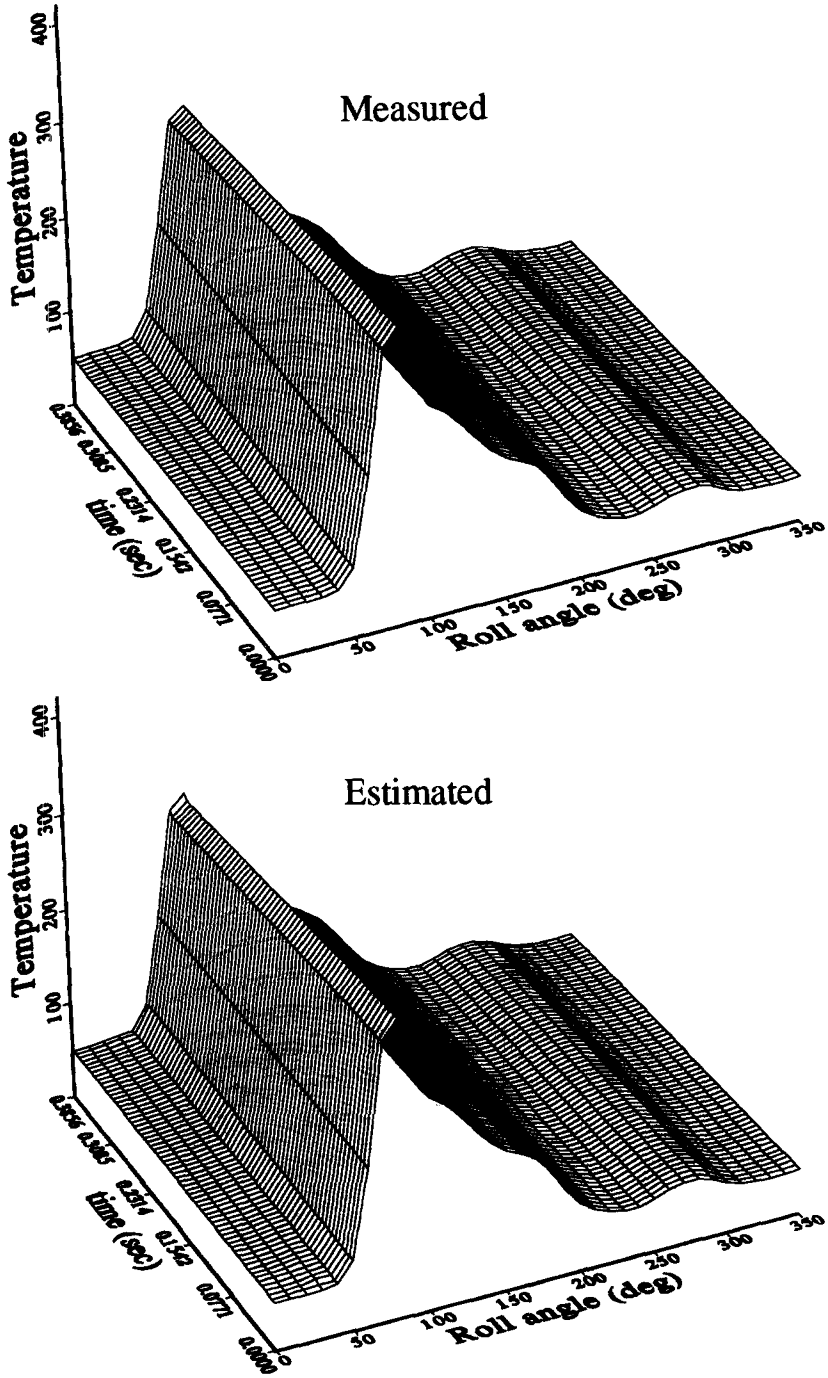


Fig. 5. Three-dimensional plot of the estimated and measured temperatures located at 0.2 mm from the roll surface for case (i).

$R = 0.355$ m, as shown in Figs. 4(b) and 6. Finally, the estimated surface heat flux of the roll and its value at the end of one cycle (i.e. $t = 0.3856$ s) are shown in Fig. 7(a) and (b), respectively. Figure 7(a) and (b) indicates that the maximum prescribed heat in the bite region is about 4×10^7 W m^{-2} at 60° roll angle, and the heat flux decreases drastically at $\theta = 35°$ due to water cooling. In addition, the heat flux should be negative outside the bite region because heat is dissipated into the atmosphere. However, the heat flux at 110° roll angle becomes positive. This is because the measured surface temperature from 100 to 110° roll angle did not decrease as expected. Such a phenomena may result from measurement errors. Once the surface heat flux is estimated, the surface heat transfer coefficient of a roll can be calculated by Fourier's law, provided that the ambient temperature and the temperature of the strip within the bite region are given.

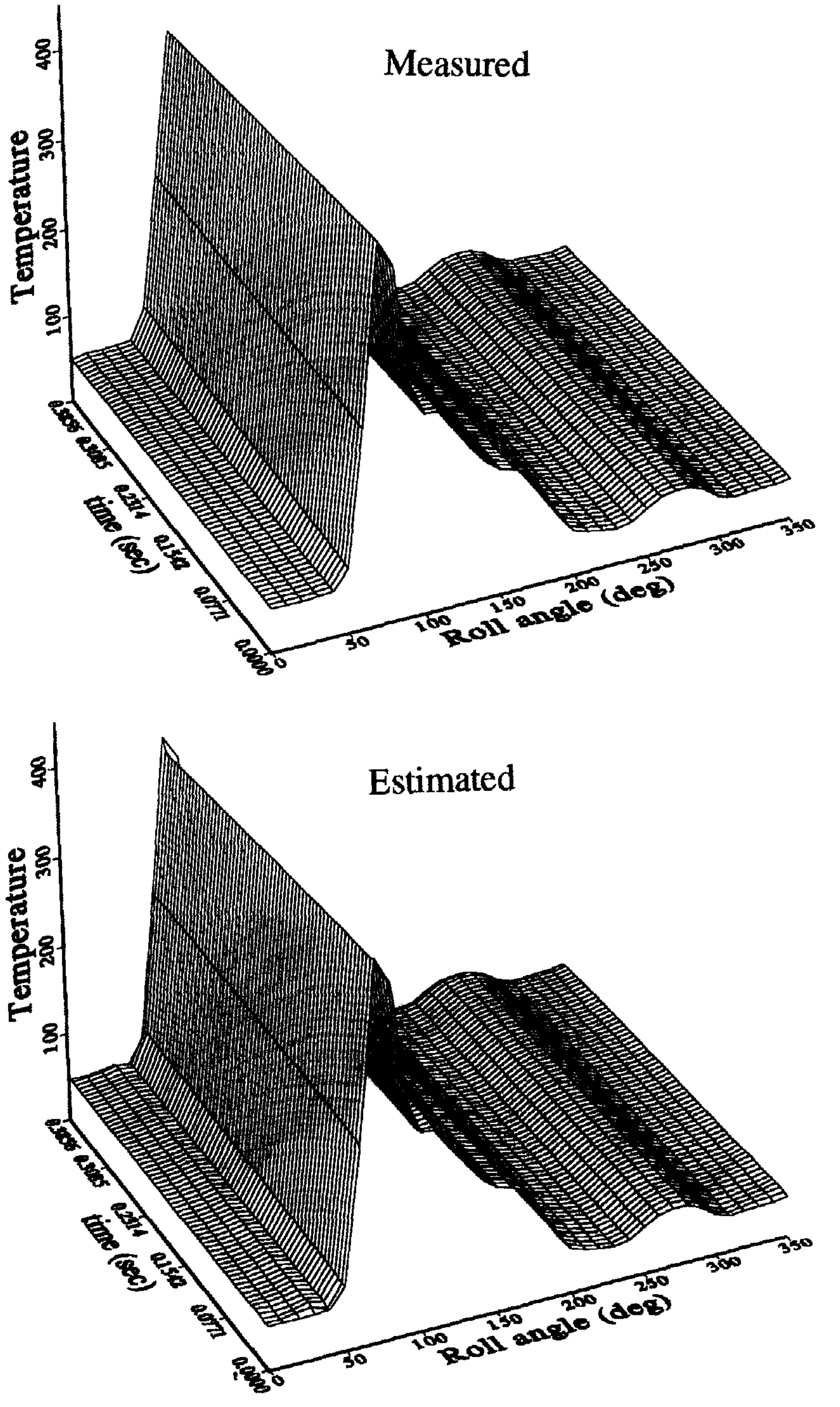


Fig. 6. Three-dimensional plot of the estimated and measured temperatures located on the roll surface for case (i).

8.2. *The measured position is* 1 *mm from the roll surface*

Figure 2 shows the geometry of the problem. The estimations of temperature and surface heat flux are shown in Figs. 8–11. From Figs. 8–11 we conclude that the estimated and measured temperatures at $R = 0.354$ m match quite well with each other, while the estimations of surface temperatures are worse than case (i). This is because the sensor in this case was placed too far away from the roll surface: thus the surface information is damped and the real surface conditions can not be recast very well. Finally the estimation of surface heat flux of a roll is given in Fig.

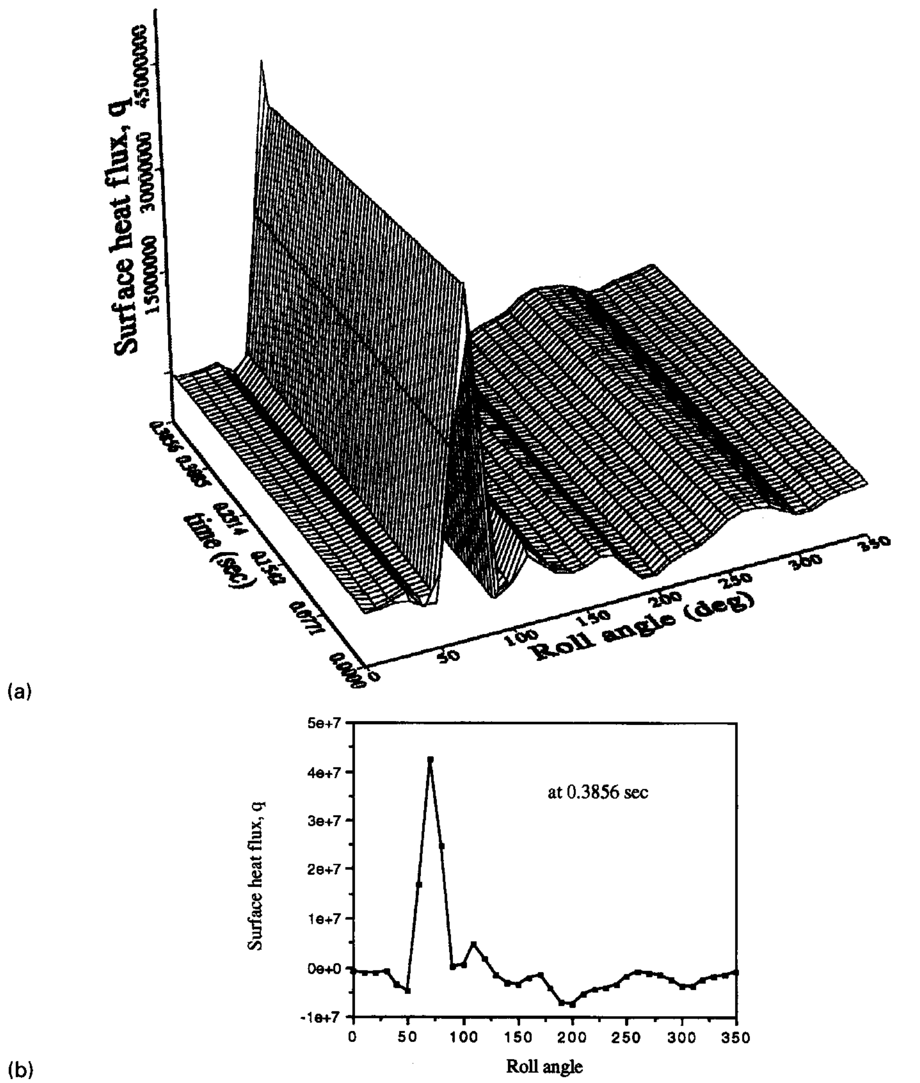


Fig. 7. (a) Three-dimensional plot of the estimated roll surface heat flux for case (i). (b) The estimated roll surface heat flux at t = 0.3856 s in (a).

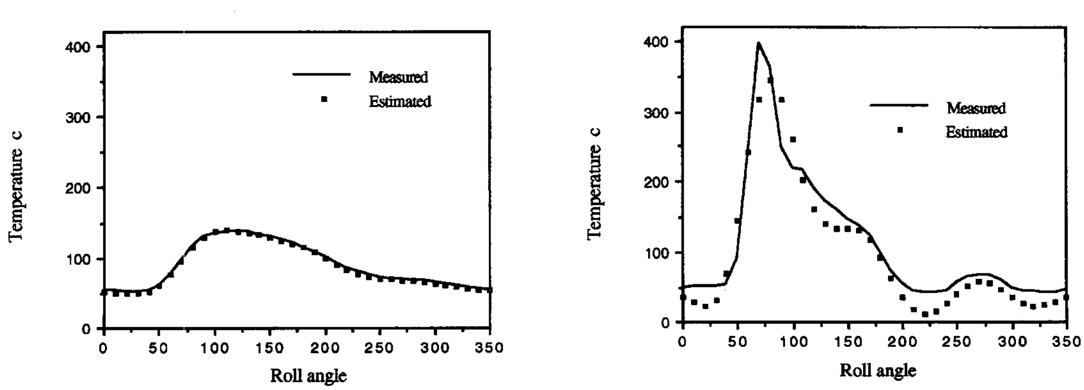


Fig. 8. (a) The estimated and measured temperatures located at 1.0 mm from the roll surface at t = 0.3856 s for case (ii). (b) The estimated and measured temperatures located on the roll surface at t = 0.3856 s for case (ii).

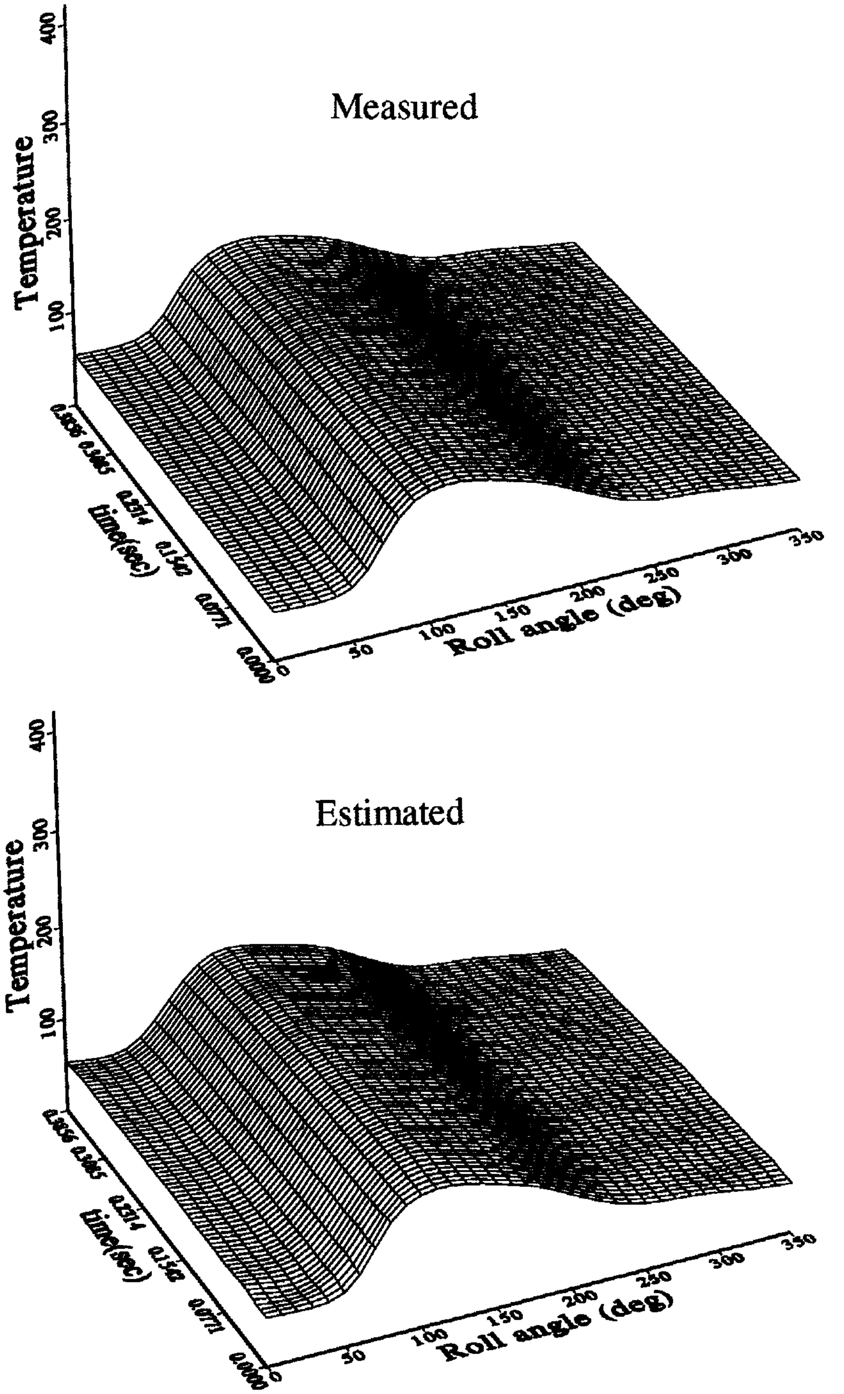


Fig. 9. Three-dimensional plot of the estimated and measured temperatures located at 1.0 mm from the roll surface for case (ii).

11, where the strange phenomena of heating at 110° roll angle did not appear due to the damping effect of the measured temperature.

From the comparison of the above two cases we found that, for all the inverse calculations performed here, the unknown values can be estimated from zero initial guess within only 35 iterations. This shows the fast convergence speed in the conjugate gradient method. The measured position has a significant effect on the inverse solutions: this is because a very thin thermal layer is formed in the transient process of roll milling.

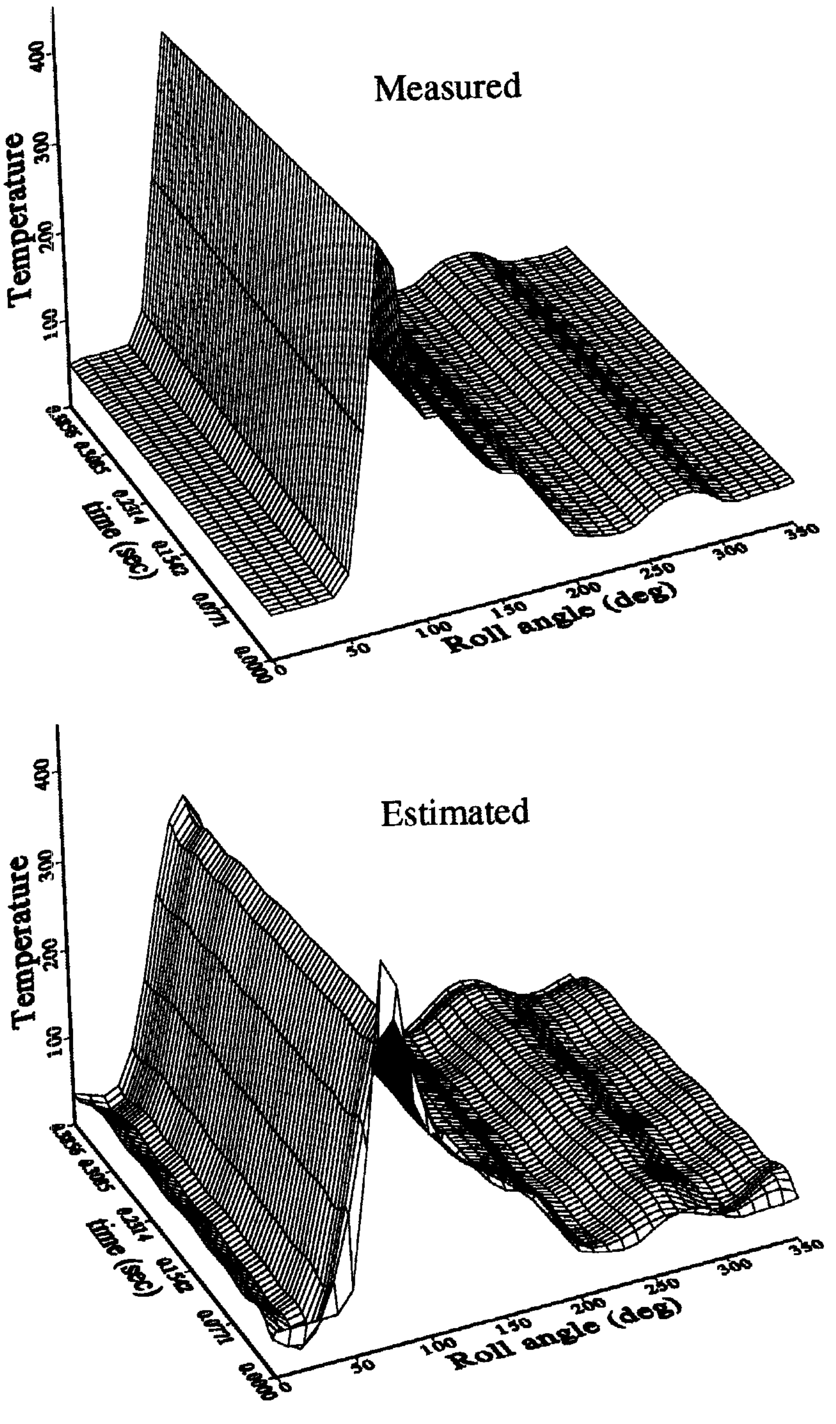
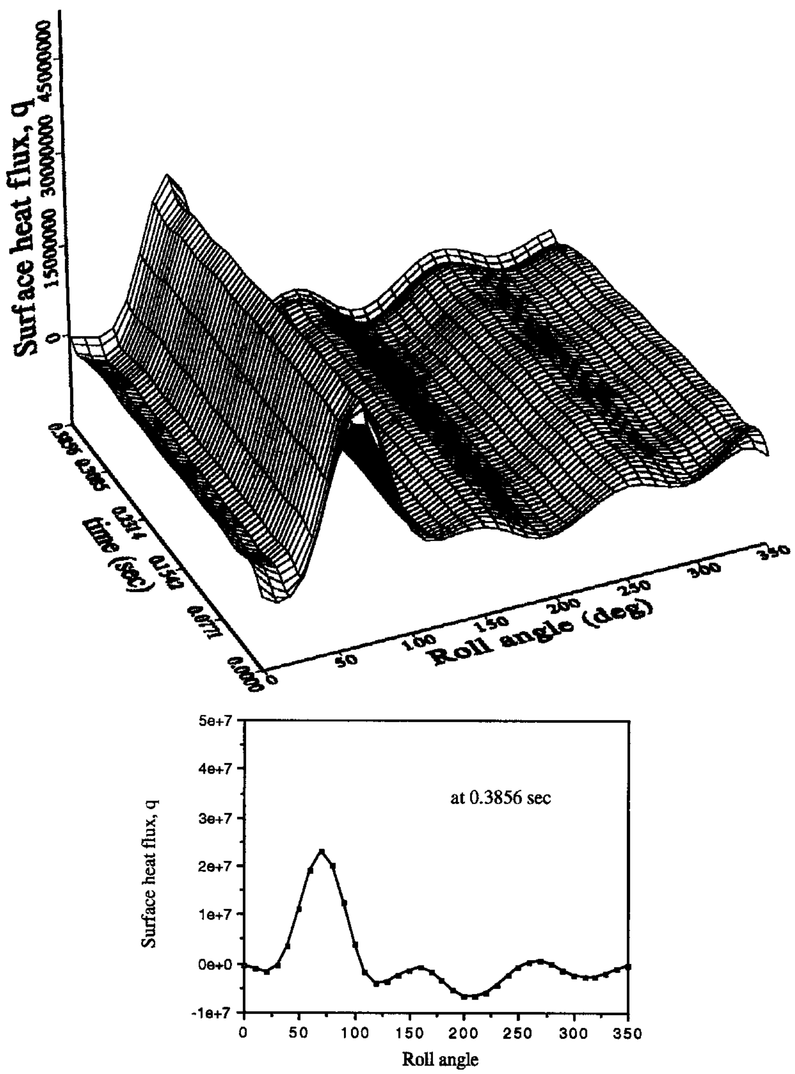


Fig. 10. Three-dimensional plot of the estimated and measured temperatures located on the roll surface for case (ii).

9. CONCLUSIONS

The conjugate gradient method with adjoint equation was successfully applied for the solution of the two-dimensional inverse-problem involving determination of the unknown surface thermal behavior of a roll. The surface heat transfer coefficient of the roll can be calculated and used in the thermal stress calculation thereafter, provided that the ambient temperature and the temperature of the strip within the bite region are known.

Comparison of the results obtained from two different temperature measured positions reveal that, in order to yield better surface conditions, the position



(a)

(b)

Fig. 11. (a) Three-dimensional plot of the estimated roll surface heat flux for case (ii). (b) The estimated roll surface heat flux at $t = 0.3856$ s in (a).

of the sensor is preferred closer to the roll surface, since the thermal layer is very thin. The results of the inverse solution show that the conjugate gradient method converges very fast (within 35 iterations) to obtain the inverse solution and does not require *prior* information on the functional form of the unknown quantities.

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REFERENCES

1. A. A. Tseng, F. H. Lin, A. S. Gunderia and D. S. Ni, Roll cooling and its relationship to roll life, *Metall. Trans.* **20A,** 2305–2320 (1989).
2. D. M. Parke and J. L. L. Baker, Temperature effects of cooling work rolls, *Iron Steel Engng* **49,** 83–88 (1972).
3. J. V. Poplawski and D. A. Seccombe Jr, Bethlehem's contribution to the mathematical modeling of cold rolling in tandem mills, *Iron Steel Engng* **57,** 47–58 (1980).
4. P. J. Roache, *Computational Fluid Dynamics.* Hermosa, Albuquerque, NM (1972).
5. J. Isenberg and G. deVahl Davis, Finite difference methods in heat and mass transfer. In *Topics in Transport Phenomena: Bioprocesses, Mathematical Treatment, Mechanism* (Edited by C. Gulfinger), pp. 457–553. Wiley, New York (1975).
6. R. M. Smith and A. G. Hutton, The numerical treatment of advection: a performence comparison of current methods, *Numer. Heat Transfer* **5,** 439–461 (1982).
7. J. V. Beck, B. Blackwell and C. R. St Clair, *Inverse Heat Conduction.... Ill-Posed Problem.* Wiley, New York (1985).
8. O. M. Alifanov, Solution of an inverse problem of heat conduction by iteration methods. *J. Engng Phys.* **26**(4), 471–476 (1974).
9. C. H. Huang and M. N. Ozisik, Inverse problem of determining unknown wall heat flux in laminar flow through a parallel plate duct, *Numer. Heat Transfer A* **21,** 55–70 (1992).
10. C. H. Huang and M. N. Ozisik, Conjugate gradient

method for determining unknown contact conductance during metal casting, *Int. J. Heat Mass Transfer* **35**, 1779–1786 (1992).

11. C. H. Huang and M. N. Ozisik, Inverse problem of determining the unknown strength of an internal plate heat source, *J. Franklin Inst.* **329**, 751–764 (1992).
12. C. H. Huang and T. M. Ju, Inverse problem of determining the periodic thermal contact conductance between exhaust valve and seat in an internal combustion engine, ASME 1993 National Heat Transfer Conference, Paper No. 93-HT-35, Atlanta, GA (8–11 August 1993).
13. O. M. Alifanov and N. V. Kerov, Determination of external thermal load parameters by solving the two-dimensional inverse heat conduction problem, *J. Engng Phys.* **41**, 581–586 (1981).
14. Y. Jarny, M. N. Ozisik and J. P. Bardon, A general method using adjoint equation for solving multi-dimensional inverse heat conduction, *Int. J. Heat Mass Transfer* **34**, 2911–2919 (1991).
15. C. H. Huang and T. M. Ju, Two dimensional inverse problem of determining the interface contact conductance between exhaust valve and seat in an internal combustion engine, *22nd International Thermal Conductivity Conference*, Tempe, AZ, pp. 985–997 (7–10 November 1993).
16. C. H. Huang and J. Y. Wu, Two-dimension inverse-problem of estimating heat fluxes inside internal combustion engines, *J. Appl. Phys.* **76** (1) (1994).
17. A. A. Tseng, Finite-difference solutions for heat transfer in a roll rotating at high speed, *Numer. Heat Transfer*, **7,** 113–125 (1984).
18. A. N. Kolmogorov and S. V. Fomin, *The Elements of Function Theory and Functional Analysis*. Nauka, Moscow (1972).